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Analysis of the magnetoresistance of three-dimensional amorphous metals with weak localization and electron–electron interaction theories

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Abstract. The magnetoresistance of three-dimensional amorphous metals with strong spin–orbit scattering is often difficult to fit to present theories of weak localization and electron–electron interaction. Especially, the fitted values are sensitive to various assumptions in the fits and only quite wide ranges for the parameters controlling the phenomena can be given. Two methods to extract the parameters in a less sensitive way are discussed. A good agreement with theory is found. The principal behaviour for the studied strong spin–orbit scattering system is identical to what is found in amorphous metals with weak spin–orbit scattering.

1. Introduction

The low-temperature behaviour of the electrical resistivity ρ and of the magnetoresistance $\Delta\rho(B) = \rho(B) - \rho(0)$ for 3D amorphous metals can be explained in terms of weak localization (WL) and electron–electron interaction (EEI) effects [1–10]. The temperature coefficient of ρ is generally negative, but in some alloy systems a shallow minimum and maximum in $\rho(T)$ can be observed. The magnetoresistance is either positive or negative with $\Delta\rho(B) \sim B^2$ for low B and $\Delta\rho(B) \sim B^{1/2}$ for high B. The magnitude of the effects is proportional to ρ, since both WL and EEI depend on a short elastic electron mean free path.

Although good overall agreement with theory can be stated, there are difficulties in obtaining consistent quantitative values for the parameters controlling $\rho(T)$ and $\Delta\rho(B)$ within the theories. In principle, it should be possible to determine the inelastic scattering time τ_{ie}, the spin–orbit scattering time τ_{so} and the electron–electron screening parameter F through

$$\rho(T) = \rho(0) + \Delta\rho(\tau_{ie}(T), \tau_{so}, \rho, D)_{WL} + \Delta\rho(F, T, \rho, D)_{EEI} \qquad (1)$$

when the electronic diffusion constant D and ρ are known. The WL part is given by Fukuyama and Hoshino [11] and the EEI part by Altshuler, Aronov and co-workers in a number of papers [1a, 12–14]. The temperature dependence of τ_{ie} is, however, not necessarily a simple power law $\tau_{ie} \sim T^{-p}$ over the studied temperature range, as it would be if only one scattering mechanism dominates. Different models for the temperature dependence of τ_{ie} give $p = 1.5$ for inelastic electron–electron scattering and $p = 2, 3$ or

4 for inelastic electron–phonon scattering [15]. One would also expect τ_{ie} to saturate when $T \to 0$ owing to either zero-point motion [16] or spin-flip scattering from small amounts of magnetic impurities [17].

It is also possible that the temperature dependence given by EEI effects, $\Delta\rho(T)_{EEI} \sim T^{1/2}$, has to be corrected with a term that weakens the temperature dependence even at such low temperatures as a few Kelvins [18]. This is in the temperature range where $\rho(T)$ can usually be interpreted in terms of a $T^{1/2}$ behaviour in amorphous alloys. Furthermore, both the contributions to $\rho(T)$ that are relevant in the discussion of the origin of the negative temperature coefficient of amorphous metals at room temperature as well as the normal Boltzmann conductivity have to be considered when $T \geqslant 10$ K. It is therefore, in practice, impossible to say anything specific about τ_{ie}, τ_{so} and F by using only equation (1).

The magnetoresistance alone or better combined with $\rho(T)$ offers a better opportunity. The contribution from classical magnetoresistance effects can be estimated to $\Delta\rho(B)/\rho \ll 10^{-8}$ [19] and is negligible. The complete contribution to $\Delta\rho(B)$ is thus given by

$$\Delta\rho(B) = \Delta\rho(\tau_{ie}(T), \tau_{so}, \rho, D, g^*, B)_{WL} + \Delta\rho(F, \rho, D, T, g^*, B)_{p\text{-}h} + \Delta\rho(g(T, B), \rho, D, T, B)_{p\text{-}p} \tag{2}$$

where $\Delta\rho_{WL}$ includes the effect of spin splitting as given by Fukuyama and Hoshino [11], $\Delta\rho_{p\text{-}h}$ is the EEI effect in the particle–hole channel given by Lee and Ramakrishnan [20] and $\Delta\rho_{p\text{-}p}$ is the EEI effect in the particle–particle channel given by Altshuler *et al* [21]. The parameters g^* and $g(T, B)$ are an effective g-factor and an electron–electron interaction constant, respectively. The fit of equation (2) to experimental data is also problematic. The EEI contributions are generally small when $\mu_B B/k_B T < 1$. Thus for fields of a few teslas the main contribution to $\Delta\rho(B)$ comes from WL ($T \geqslant 4$ K assumed). In strong spin–orbit scattering systems, i.e. $\tau_{so} < \tau_{ie}$ and $\Delta\rho(B) > 0$, the magnitude of $\Delta\rho(B)$ is often found to be larger than what is expected from theory. In the fittings this may appear as a τ_{ie} rapidly approaching infinity as T is decreased. One also has to suspect that a strong temperature dependence, such as $\tau_{ie} \sim T^{-3}$, down to $T \simeq 4$ K has the same origin rather than a true physical behaviour of τ_{ie}, since in weak spin–orbit scattering systems, $\tau_{so} > \tau_{ie}$ and $\Delta\rho(B) < 0$, τ_{ie} shows a weakening in the temperature dependence starting already at 10 K and saturation in τ_{ie} as $T \to 0$ [7, 10, 22]. At higher temperatures $\tau_{ie} \sim T^{-p}$ with p varying between 2 and 4 for different alloys, indicating electron–phonon scattering as the main inelastic scattering mechanism.

Trudeau and Cochrane [23] have proposed that in nearly magnetic alloys the spin splitting in the WL expression is enhanced with the Stoner enhancement factor $(1 - I)^{-1}$, thus $g^* \to g^*/(1 - I)$. The maximum value of $\Delta\rho(B)$ within the WL contribution becomes most important at the lowest temperatures. If these temperatures can be fitted, it is usually no problem to fit $\Delta\rho(B)$ at higher T. The problem of an apparent temperature dependence of τ_{so} found by Hickey *et al* [7] may be solved by introducing the Stoner enhancement factor. Thus usually means that g^* is a fitting parameter as well.

When all these aspects have been considered, the allowed ranges for τ_{ie} and τ_{so} are wide and the information on the EEI parameters F and $g(T, B)$ is lost in most cases.

Before one applies the theory to data one would like to simplify the problem as much as possible. This can be done first of all by selecting samples that are neither superconducting nor magnetic. The contribution from superconducting fluctuations well above the superconducting transition temperature and scattering from large amounts of coupled spins can then be neglected. Secondly, one can study a limit where the theories become simpler or use special features in the data.

In this paper two methods will be investigated. The first suggests the possibility of a separation of $\Delta\rho(B)_{\mathrm{EEI}}$ from $\Delta\rho(B)$ before fitting to theory. The result is compared with the result from $\rho(T)$. The second method uses the B^2 range of $\Delta\rho(B)$ to determine $\tau_{\mathrm{ie}}(T)$ and τ_{so} on $Cu_{65}Ti_{35}$ over an extended temperature range. In this regime $\Delta\rho(B) = \alpha(\tau_{\mathrm{ie}}, \tau_{\mathrm{so}})B^2$ instead of $\Delta\rho(B) = \beta(\tau_{\mathrm{ie}}, \tau_{\mathrm{so}}, B)$. To reach this limit a resolution of about 10^{-7} in the measurements is necessary. Another advantage with this method is the absence of EEI contributions in $\Delta\rho(B)$ at low T.

2. Theoretical expressions

The temperature-dependent correction to ρ from weak localization is [11]

$$\Delta\rho(T)_{\mathrm{WL}}/\rho = \rho A[t^{1/2} - 3(t+1)^{1/2}] \tag{3a}$$

with

$$t = \tau_{\mathrm{so}}/4\tau_{\mathrm{ie}} \tag{3b}$$

$$A = (e^2/2\pi^2\hbar)(D\tau_{\mathrm{so}})^{-1/2} \tag{3c}$$

and from electron–electron interactions we have [1a, 12–14]

$$\Delta\rho(T)_{\mathrm{EEI}}/\rho = -\rho G_3(e^2/2\pi^2\hbar)(1.294/\sqrt{2})(k_{\mathrm{B}}T/\hbar D)^{1/2} \tag{4a}$$

with

$$G_3 = \tfrac{1}{2}(\tfrac{4}{3} - \tfrac{3}{2}\tilde{F}_\sigma) \tag{4b}$$

$$\tilde{F}_\sigma = (32/3F)[(1 + \tfrac{1}{2}F)^{3/2} - 1 - \tfrac{3}{4}F] \tag{4c}$$

where F is the Coulomb potential averaged over the Fermi surface. Equation (3a) has a maximum for $t = \tfrac{1}{8}$.

The magnetic field dependence due to weak localization including spin splitting [11] is

$$\frac{\Delta\rho(B)_{\mathrm{WL}}}{A\rho^2} = -h^{1/2}f_3\left(\frac{1+t}{h}\right) - \tfrac{1}{2}\left(\frac{h}{1-\gamma}\right)^{1/2}\left[f_3\left(\frac{t_+}{h}\right) - f_3\left(\frac{t_-}{h}\right)\right]$$
$$+ \frac{t_-^{1/2} - t_+^{1/2}}{(1-\gamma)^{1/2}} + (t+1)^{1/2} - t^{1/2} \tag{5a}$$

with

$$h = eDB\tau_{\mathrm{so}}/\hbar \tag{5b}$$

$$\gamma = (g^*\mu_{\mathrm{B}}h/2eD)^2 \tag{5c}$$

$$t_\pm = t + 0.5[1 \pm (1-\gamma)^{1/2}] \tag{5d}$$

$$f_3(x) = \begin{cases} 0.6049 & \text{when } x = 0 \\ x^{-3/2}/48 & \text{when } x \gg 1. \end{cases} \tag{5e}$$

The parameters t and A are defined by equations (3b) and (3c) respectively. The function

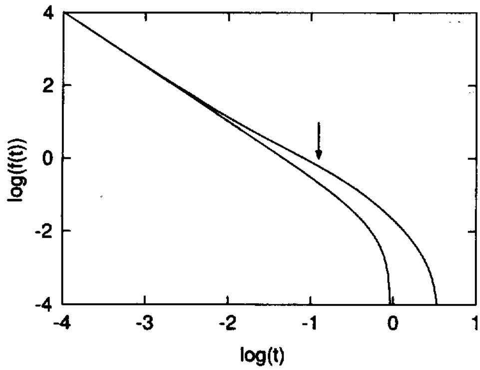

Figure 1. The function $f(t)$ (equation (6b)) for (from left) $g^* = 0$ and $g^* = g_{\mathrm{FE}} = 2$ versus t on double log scale. The figure shows the effect of adding spin splitting in weak localization. At small values of t (low T) there is almost no difference in the curves, but at large t (high T) it could be considerable. The arrow indicates the maximum in $\Delta\rho(T)_{\mathrm{WL}}$ at $t = \frac{1}{8}$. The curves are calculated with $D = 0.315\ \mathrm{cm^2\,s^{-1}}$ valid for $Cu_{65}Ti_{35}$.

$f_3(x)$ is the Kawabata function [24]. The expression is valid for all γ. The spin–orbit scattering time is defined as an isotropic lifetime $\tau_{\mathrm{so}} = (\tau_{\mathrm{so}})_x = (\tau_{\mathrm{so}})_y = (\tau_{\mathrm{so}})_z$. Redefining it as a global lifetime gives $\tau_{\mathrm{so}} \rightarrow 3\tau_{\mathrm{so}}$ [25]. If the diffusion constant $D \gg 1\ \mathrm{cm^2\,s^{-1}}$, the spin splitting can be neglected [9] by setting $g^* = 0$. For amorphous metals D is often lower than $1\ \mathrm{cm^2\,s^{-1}}$ and the spin splitting has to be included.

For low magnetic fields equation (5a) can be simplified to

$$\Delta\rho(B)_{\mathrm{WL}}/\rho = \rho(e^4/2\pi^2\hbar^3)(D\tau_{\mathrm{so}})^{3/2} f(t) B^2 \qquad (6a)$$

with

$$f(t) = \tfrac{1}{96}[t^{-3/2} - 3(t+1)^{-3/2}] + (\gamma'/8)[t^{-1/2} + (t+1)^{-1/2}] + (\gamma'/2)[t^{1/2} - (t+1)^{1/2}] \qquad (6b)$$

$$\gamma' = (g^*\mu_{\mathrm{B}}/2eD)^2. \qquad (6c)$$

A plot of $f(t)$ is shown in figure 1.

The electron–electron interaction in the particle–hole channel gives [20]

$$\Delta\rho(B)_{\mathrm{p\text{-}h}}/\rho = \rho \frac{e^2}{4\pi^2\hbar} H_3 \left(\frac{k_{\mathrm{B}}T}{2\hbar D}\right)^{1/2} g_3\left(\frac{g^*\mu_{\mathrm{B}}B}{k_{\mathrm{B}}T}\right) \qquad (7a)$$

with

$$H_3 = F \qquad (7b)$$

$$g_3(x) = \begin{cases} x^{1/2} - 1.294 & \text{when } x \gg 1 \\ 0.056\,x^2 & \text{when } x \ll 1 \end{cases} \qquad (7c)$$

and the corresponding contribution in the particle–particle channel can be written as [21]

$$\Delta\rho(B)_{\mathrm{p\text{-}p}}/\rho = \rho g(T,B) \frac{e^2}{2\pi^2\hbar}\left(\frac{eB}{\hbar}\right)^{1/2} \varphi_3\left(\frac{2DeB}{\pi k_{\mathrm{B}}T}\right) \qquad (8a)$$

with

$$\varphi_3(x) = \begin{cases} 1.900 - 2.294/x^{1/2} & \text{when } x \gg 1 \\ 0.329\,x^2 & \text{when } x \ll 1 \end{cases} \qquad (8b)$$

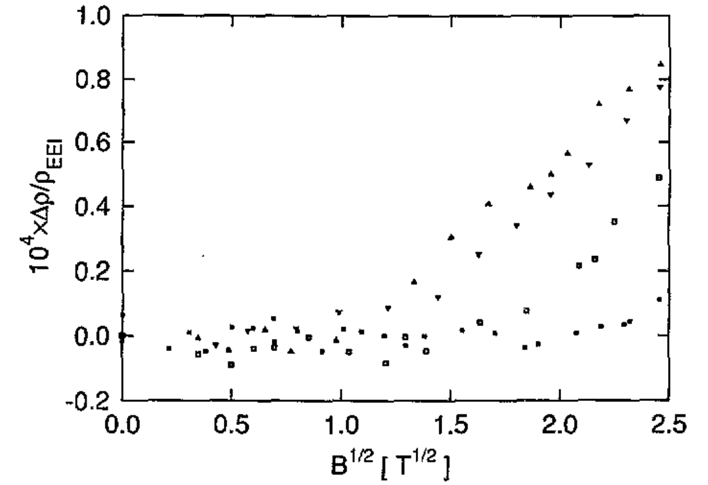

Figure 2. The EEI contribution to the magnetoresistance $\Delta\rho(B)_{EEI}/\rho$ versus $B^{1/2}$ for $Ca_{70}Al_{15}Mg_{15}$. The separation of $\Delta\rho(B)_{EEI}/\rho$ from $\Delta\rho(B)/\rho$ is described in the text. Symbols: ($\triangle$) 0.27 K, (∇) 0.68 K, ($\square$) 1.48 K, ($\blacksquare$) 4.24 K.

with $|g(T, B)| < 0.2$ for most metals. The temperature and magnetic field dependence is weak and is assumed here to be constant, $g(T, B) = g$. Alternative expressions of this term [1a, 26] have been compared and discussed by Baxter *et al* [27, 28]. For simplicity $\tilde{F}_\sigma$ (in equation (4*c*)) will be used when $\rho(T)$ is considered and F (in equation (7*b*)) for $\Delta\rho(B)$.

Numerically convenient expressions for the functions $f_3(x)$, $g_3(x)$ and $\varphi_3(x)$ have been given by Ousset *et al* [29, 30].

3. Separation of weak localization and electron–electron interaction effects

Instead of determining the EEI constants F and g by making a full fit to WL and EEI effects, one can first try to separate $\Delta\rho(B)$ into $\Delta\rho(B)_{WL}$ and $\Delta\rho(B)_{EEI} = \Delta\rho(B)_{p\text{-}h} + \Delta\rho(B)_{p\text{-}p}$ and then only fit the EEI part. The separation is reduced to subtracting a temperature-dependent parameter, by assuming that $\Delta\rho(B)_{EEI}$ can be neglected when $\mu_B B/k_B T < 1$ and that $\Delta\rho(B)_{WL}$ has a weak temperature dependence. The latter condition means that curves of $\Delta\rho(B)_{WL}$ are parallel above some low value of B. The $Ca_{70}(Al,Mg)_{30}$ system was shown recently to be the only 3D amorphous metal system so far known where the observed $\Delta\rho(B)$ is in excellent agreement with theory down to 0.1 K [31, 32]. Figure 2 shows $\Delta\rho_{EEI}$ for $Ca_{70}Mg_{15}Al_{15}$ obtained by first subtracting $\Delta\rho(B)/\rho$ at 4.2 K from all other $\Delta\rho(B)/\rho$ and then making a small correction for the presence of EEI effects in $\Delta\rho(B)/\rho$ at 4.2 K for $B > 4$ T. This can be made by demanding $\Delta\rho(B)_{EEI} \sim cB^{1/2}$ and c temperature-independent when $\mu_B B/k_B T > 1$. The temperature-dependent parameter from WL can be taken to be constant at $T < 4$ K for this alloy and, in particular, τ_{ie} = constant. Figure 3 shows $\Delta\rho(B, T)/\rho - \Delta\rho(B, 4.2\,\mathrm{K})/\rho$ for $Ca_{70}Al_{30}$. The WL effect is here not saturated. Both alloys show clearly the presence of EEI, which otherwise normally can only be observed by the $T^{1/2}$ behaviour of $\rho(T)$.

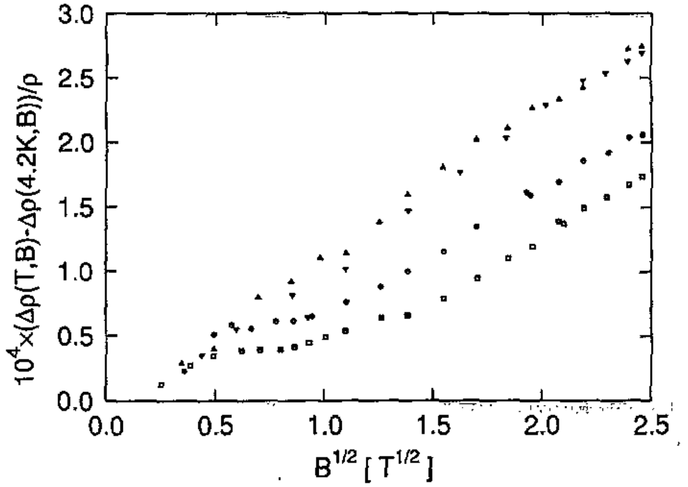

Figure 3. The difference between $\Delta\rho(T, B)/\rho$ and $\Delta\rho(4.2\,K, B)/\rho$ versus $B^{1/2}$ for $Ca_{70}Al_{30}$. Symbols: ($\triangle$) 0.18 K, (∇) 0.38 K, ($\bigcirc$) 0.78 K, ($\square$) 1.43 K.

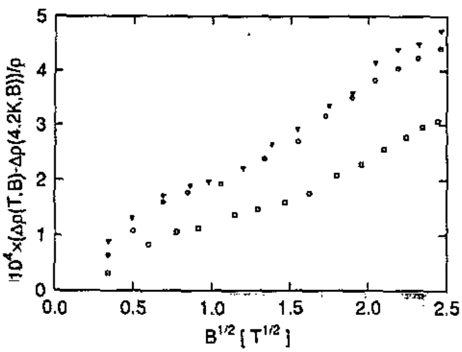

Figure 4. The difference between $\Delta\rho(T, B)/\rho$ and $\Delta\rho(4.2\,K, B)/\rho$ versus $B^{1/2}$ for $Cu_{45}Ti_{55}$. The WL contribution has a stronger temperature dependence in this alloy than in Ca(Al,Mg) as seen in $[\Delta\rho(T, B) - \Delta\rho(4.2\,K, B)]/\rho$ below 1 T. Symbols: (∇) 0.11 K, ($\bigcirc$) 0.55 K, ($\square$) 1.5 K.

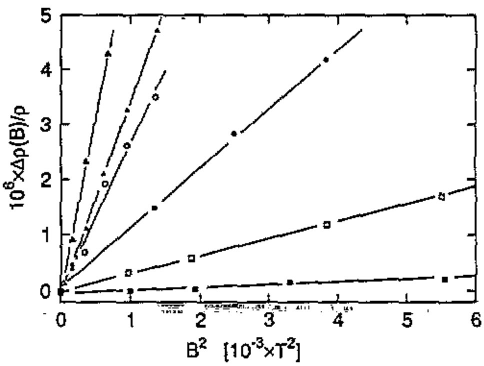

Figure 5. The $\Delta\rho(B)/\rho$ versus B^2 plot for $Cu_{65}Ti_{35}$. The straight lines are fits to $\Delta\rho(B) = \alpha B^2$. Symbols: ($\triangle$) 1.48 K, ($\blacktriangle$) 2.16 K, ($\bigcirc$) 2.88 K, ($\bullet$) 4.21 K, ($\square$) 7.59 K, ($\blacksquare$) 12.8 K.

The same method can be applied to $\Delta\rho(T, B)$ of CuTi alloys by re-examining data from [33]. Data for $\Delta\rho(T, B)/\rho - \Delta\rho(4.2\,K, B)/\rho$ for $Cu_{45}Ti_{55}$ are shown in figure 4. The contribution from WL has a stronger temperature dependence in this sample. For $Cu_{60}Ti_{40}$ and $Cu_{65}Ti_{35}$ the data have a higher noise level, but the principle behaviour of $\Delta\rho(T, B)/\rho - \Delta\rho(4.2\,K, B)/\rho$ is identical. Results of fits using equations (7) and (8) are listed in table 1. Data for all temperatures were fitted simultaneously. The effective g-factor g^* is assumed to be the free-electron value $g_{FE} = 2$. The value of g^* is briefly discussed in section 4.

The $\tilde{F}_\sigma$ parameter can be obtained from $\rho(T)$ at low temperatures from equation (1) if the WL contribution is known. For the Ca(Mg,Al) alloys the temperature dependence

Table 1. Sample properties and EEI parameters obtained in fits to and calculations from theory.

Sample	ρ ($\mu\Omega$ cm)	D^{a} ($\mathrm{cm^2\,s^{-1}}$)	$-\frac{1}{\rho}\frac{d\rho^{b}}{d(T^{1/2})}$ ($10^{-4}\,\mathrm{K^{-1/2}}$)	g^{c}	F^{c}	F^{d}	$\tilde{F}_\sigma^{e}$	$\tilde{F}_\sigma$ (calc)^{f}
$Ca_{70}Mg_{15}Al_{15}$	126 ± 4	2.05	1.35 ± 0.1	−0.018	0.41	0.26	0.39 ± 0.05	0.65
$Ca_{70}Al_{30}$	312 ± 7	0.97	5.66 ± 0.1	−0.001	0.25	0.24	0.31 ± 0.03	0.60
$Cu_{45}Ti_{55}$	195 ± 5	0.250	10.1 ± 0.5	−0.007	0.36	0.31	< 0.08	0.82
$Cu_{60}Ti_{40}$	187 ± 5	0.285	6.45 ± 0.3	–	–	0.4*	< 0.32	0.82
$Cu_{65}Ti_{35}$	182 ± 5	0.315	5.30 ± 0.3	–	–	0.5*	0.24–0.40	0.82

[a] Calculated from specific-heat measurements in [36] and [38].
[b] For Ca(Al,Mg), $0.2 < T < 4.2$ K; and for CuTi, $0.25 < T < 1$ K.
[c] g and F fitted.
[d] F fitted with $g = 0$; *for $Cu_{60}Ti_{40}$ and $Cu_{65}Ti_{35}$ F is calculated from $\Delta\rho(B)_{\mathrm{EEI}} = aB^{1/2}$.
[e] $\tilde{F}_\sigma$ as calculated from $\rho(T)$.
[f] Theoretical values calculated as described in the text.

of τ_{ie} is weak or absent. The observed temperature dependence of ρ thus mainly stems from EEI. Values are listed in table 1. For $Cu_{65}Ti_{35}$ we extrapolate $\tau_{ie} = 6.0 \times 10^{-10}/T$ for $T < 1.5$ K and τ_{so} at most of the order of some picoseconds (see section 4). When $\tau_{ie} \gg \tau_{so}$ the temperature-dependent part of equation (3) is approximated by

$$\Delta\rho(T)/\rho = \rho\,(e^2/4\pi^2\hbar)(D\tau_{ie})^{-1/2}. \tag{9}$$

From equations (4) and (9) $\tilde{F}_\sigma$ can be calculated. This gives $\tilde{F}_\sigma = 0.28 \pm 0.04$. The extrapolated temperature dependence of τ_{ie} can be too strong for $0.3\,\mathrm{K} < T < 1.0\,\mathrm{K}$, which may underestimate $\tilde{F}_\sigma$. Setting τ_{ie} = constant gives the upper limit of $\tilde{F}_\sigma = 0.40$.

A free-electron model and the Thomas–Fermi approximation give

$$F = \ln(1+\chi)/\chi \tag{10a}$$

with

$$\chi = (4\pi^4\hbar^4\varepsilon_0/e^2m^2)N(0) \tag{10b}$$

where $N(0)$ is the electronic density of states and ε_0 the dielectric constant. Equations (4*c*) and (10) yield $0 \leqslant \tilde{F}_\sigma \leqslant 0.93$ for $0 \leqslant F \leqslant 1$. An approximation to equation (4*c*) is $\tilde{F}_\sigma = 0.9F$. Free-electron values for $N(0)$ give $0.45 \leqslant \tilde{F}_\sigma \leqslant 0.55$ for most amorphous metals. The density $N(0)$ as calculated from the specific heat and a corresponding enhancement of the electron mass over the free-electron mass gives higher values for $\tilde{F}_\sigma$ as given in table 1.

The values for $\tilde{F}_\sigma$ and F as determined from experimental data are much lower than the calculated one. The agreement between F from $\Delta\rho(B)$ and $\tilde{F}_\sigma$ from $\rho(T)$ is good, with $Cu_{45}Ti_{55}$ as an exception. The contribution to $\rho(T)$ from the particle–particle channel has not yet been considered. Doing this, we get [26]

$$G_3 = \tfrac{1}{2}\{\tfrac{4}{3} - \tfrac{3}{2}\tilde{F}_\sigma - F/[1 + \tfrac{1}{2}F\ln(1.13\omega_0/T)]\} \tag{11}$$

where ω_0 is the Fermi temperature T_F or the Debye temperature θ_D in the case of a superconductor. For a superconductor G_3 will reduce to

$$G_3 = \tfrac{1}{2}[\tfrac{4}{3} - \tfrac{3}{2}\tilde{F}_\sigma - 2/\ln(T_c/T)]. \tag{12}$$

A non-superconductor gives a small negative contribution from the last term in equation

(11) of 0.06–0.09 for $0.2 < F < 0.5$. The obtained values of $\tilde{F}_\sigma$ are slightly overestimated. If we use Isawa and Fukuyama's interaction constant for the magnetic field contribution from the particle–hole channel [26],

$$H_3 = -4 - (8/F)\ln(1 - F/2) \tag{13}$$

we find F to be overestimated by 10% for $Ca_{70}Al_{30}$ and 16% for $Cu_{65}Ti_{35}$. For small F this expression reduces to F.

There are a number of assumptions in the calculations of F and $\tilde{F}_\sigma$ from experimental data. A 100% agreement is therefore not expected and the overall agreement in table 1 is thus surprisingly good. The use of values calculated from $\rho(T)$ to calculate $\Delta\rho(B)_{\mathrm{EEI}}$ and vice versa is dangerous. The obtained values can only be interpreted as rough estimates. Theoretical calculations from the Thomas–Fermi approximation generally give too large values and are not valid when the effective electron–electron interaction is positive. In this case F should be negative. In the alloys studied here, g is small and of minor importance. The small negative values obtained in the fits may only be a result of small systematic errors.

4. Evaluation of τ_{ie} and τ_{so} from measurements of $\Delta\rho(B)$ in the low-B limit in $Cu_{65}Ti_{35}$

$Cu_{65}Ti_{35}$ is a strong spin–orbit scattering system ideal for a study of $\Delta\rho(B)$ in the low-B limit. The resistivity $\rho(B, T)$ and other electronic properties of CuTi alloys have been studied by a number of groups [3, 4, 9, 33, 34–36] and are well documented.

4.1. *Experimental method*

The magnetoresistance of amorphous $Cu_{65}Ti_{35}$ (sample preparation is described in [9]) was measured in a wide temperature range from 1.5 to 273 K. The magnitude of $\Delta\rho(B)/\rho$ at maximum magnetic field was of the order 10^{-6}–10^{-5} with an RMS value of about 5×10^{-8} for fitted B^2 lines below 40 K and about one order of magnitude larger at higher temperatures. The high resolution in the resistance measurement was achieved with an eight-decade Guideline 9970 resistance DC bridge with an ultimate sensitivity better than 0.5 nV. Below 50 K an Allen–Bradley carbon resistor was used for temperature control. Corrections were made for temperature errors due to magnetoresistance of the thermometer corresponding to additive corrections to $\Delta\rho(B)/\rho$ rising from 5 to 30% in the range 20 to 50 K. Above 50 K the sample was measured in liquid gases at the boiling points and in water at the ice point. In these latter measurements a Pt resistor was used and the sample was measured at increasing as well as decreasing field. The data were corrected for the temperature drift of about 100–300 mK over a measurement cycle. The error in the slopes obtained around 80 K is estimated to be 10–20% and at 273 K to be 50–100%. Several results are shown in figures 5 and 6.

4.2. *Results*

The slopes $\alpha = \Delta\rho(B)/\rho\, B^2$ are shown in figure 7. As seen, the temperature dependence of α below 10 K is quite weak, $\alpha \sim T^{-1.5}$; for $10\ \mathrm{K} < T < 50\ \mathrm{K}$ strong, $\alpha \sim T^{-5.5}$; and for $T > 50$ K weak again.

The theoretical expressions to be considered are equations (6), (7) and (8). The EEI effect in the particle–particle channel is usually much smaller than the contribution from

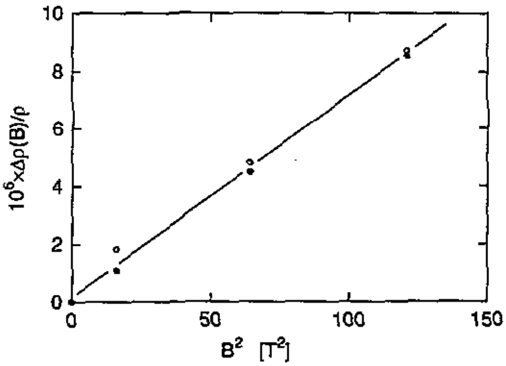

Figure 6. The $\Delta\rho(B)/\rho$ versus B^2 plot for $Cu_{65}Ti_{35}$ at 85 K. The figure shows two series of measurements after compensation for temperature drifts during the measurement. The straight line is a fit to $\Delta\rho(B) = \alpha B^2$.

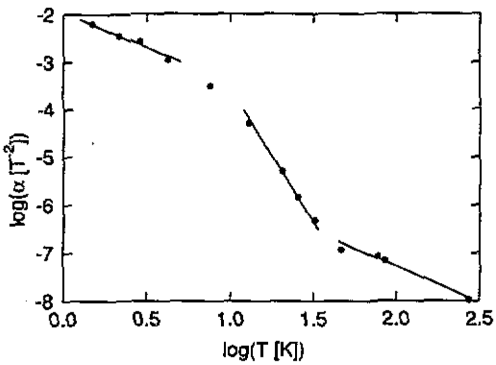

Figure 7. The slopes $\alpha = \Delta\rho(B)/\rho B^2$ versus temperature on double log scale for $Cu_{65}Ti_{35}$. The straight lines give the approximate strength of the temperature dependence in three different temperature ranges: $1.5\,\mathrm{K} < T < 5\,\mathrm{K}$, $\alpha \sim T^{-1.5}$; $20\,\mathrm{K} < T < 40\,\mathrm{K}$, $\alpha \sim T^{-5.5}$; $50\,\mathrm{K} < T < 273\,\mathrm{K}$, $\alpha \sim T^{-1.5}$. The last value is uncertain.

the particle–hole channel when $\Delta\rho(B) \sim B^2$, i.e. $\Delta\rho(B)_{\text{p-p}} \leqslant 0.1\,\Delta\rho(B)_{\text{p-h}}$, and can be neglected. Below about 20 K $\Delta\rho(B)_{\text{p-h}}$ can also be neglected. From equations (6) and (7) we get

$$\alpha(T) = \rho(e^4/2\pi^2\hbar)(D\tau_{so})^{3/2}f(t) + 0.056\,\rho(e^2/4\pi^2\hbar)F(2\hbar D)^{-1/2}(g^*\mu_B)^2(k_BT)^{-3/2}. \tag{14}$$

This expression, however, cannot be fitted, since $f(t)$ has most likely a non-trivial temperature dependence. The parameter t can only be calculated for an assumed F and τ_{so}.

One possibility is to use the temperature dependence of ρ. It has a maximum at $T \simeq 10$ K [9], which can be identified with the maximum in equation (3). However, EEI effects are also present in $\rho(T)$, with $\mathrm{d}\rho_{\text{EEI}}/\mathrm{d}T < 0$. The maximum from WL effects is therefore at higher temperatures. For different assumptions about this maximum at temperature T_{max} ($t = \frac{1}{8}$) we get different values for τ_{so} as shown in figure 8. This is based on different assumptions for $\tilde{F}_\sigma$ in equation (4). From $\Delta\rho(T)$ below 1 K, one gets $\tilde{F}_\sigma \leqslant 0.40$ [9]. This gives $T_{max} > 18$ K. From section 3 we have $F \simeq 0.5$ giving $\tau_{so} = 1.6$ ps when $g^* = 2$ and $T_{max} = 17$ K. Depending on the assumptions for T_{max} and $\tilde{F}_\sigma(F)$ we find for p in $\tau_{ie} \sim T^{-p}$ a value of 1.15 ± 0.1 when $1.5\,\mathrm{K} < T < 5\,\mathrm{K}$ and 3.7 ± 0.5 when $20\,\mathrm{K} < T < 50\,\mathrm{K}$. It is not possible to give definite values for all τ_{ie}, since the EEI may have weakened and the exact value of g^* is unknown. The temperature dependences given here could be seen as approximations over the given temperature ranges. The data are consistent with a τ_{ie} saturating when $T \to 0$, as has been found in weak spin–orbit scattering systems [10, 31]. At 1.5 K the scattering time $\tau_{ie} = 400$ ps is independent of different assumptions. The scattering time τ_{ie} can be described by a combination of saturation, electron–electron scattering and electron–phonon scattering

$$1/\tau_{ie} = A_0 + A_1 T^{3/2} + A_2 T^n \tag{15}$$

with $3 \leqslant n \leqslant 4$. The result of such a fit is shown in figure 9.

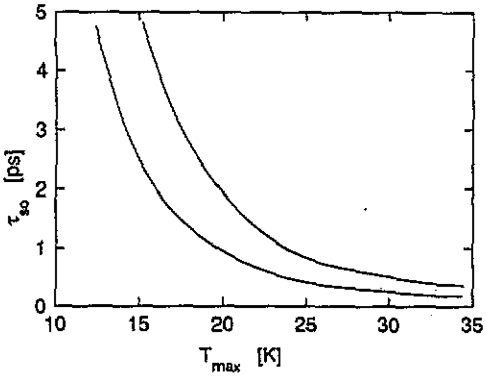

Figure 8. The spin–orbit scattering time τ_{so} versus T_{max} for $Cu_{65}Ti_{35}$. The upper curve is for $g^* = 0$ and the lower for $g^* = 2$ (the free-electron value g_{FE}). (T_{max} is the temperature for which $t = \frac{1}{8}$; see the text.) The figure illustrates the strong interrelation between τ_{so} and the assumed T_{max} in the fitting procedures.

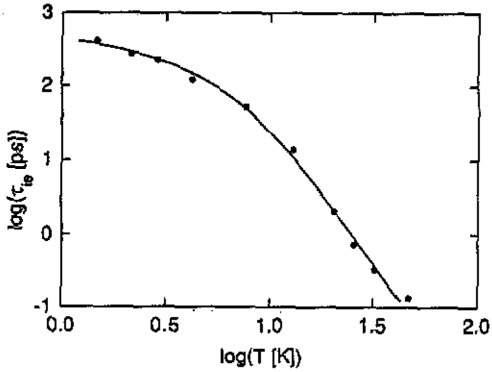

Figure 9. The inelastic scattering time τ_{ie} versus T on double log scale for $Cu_{65}Ti_{35}$. It was assumed that $g^* = g_{FE} = 2$ and $\tilde{F}_\sigma = 0.3$. The full curve is a fit to equation (15) with $n = 4$. The fit gives a saturation value of $\tau_{ie} = 560$ ps.

One can also fit equations (3) and (4) with τ_{so} and τ_{ie} calculated from experimental values of $\Delta\rho(B)$ and equations (6) and (7). This gives

$$\rho(T)_{exp} = \rho(0) + \Delta\rho(T)_{EEI} + \Delta\rho(\Delta\rho(B)_{exp}, \Delta\rho(B)_{WL}, \Delta\rho(B)_{EEI})_{WL} \tag{16}$$

with $\rho(0)$, $\tilde{F}_\sigma(F)$ and g^* as fitting parameters. This gives poor fits when data for which $T \leqslant 50$ K are used. This is not surprising since several other effects, as has been discussed may contribute to $\rho(T)$. The maximum and minimum in $\rho(T)$ can only be reproduced when data for $T \leqslant 20$ K are used and a good fit is only obtained when $T < 15$ K (six points). The maximum in $\Delta\rho(T)_{WL}$ is at $12\text{ K} \leqslant T_{max} \leqslant 20$ K in all fits. If one assumes that the EEI effect is not weakened and that $\Delta\rho(B)_{EEI} \gg |\Delta\rho(B)_{WL}|$ at $T \simeq 85$ K, then it is possible to determine F and g^*/g_{FE} by combining $\Delta\rho(B, 85\text{ K})$ and the low-temperature $\Delta\rho(B)_{EEI}$ from section 3. This gives $F = 0.4 \pm 0.05$ and $g^*/g_{FE} = 1.65 \pm 0.2$. The result is interesting, since it is the first attempt to determine g^*/g_{FE} from $\Delta\rho(B)$. It supports the proposal that g^* may be enhanced with a Stoner enhancement factor, although the present value would seem to be rather large for a CuTi alloy. Further investigations examining the assumptions of this analysis are needed to make a more definite statement. An enhancement of g^* will give a smaller value for τ_{so} and a stronger temperature dependence of τ_{ie}.

There are a few attempts to make similar analyses from the B^2 region of the magnetoresistance of amorphous alloys. Schulte and Fritsch found $2 < p < 3$ over a more limited temperature range ($6\text{ K} < T < 20$ K) for a number of CuTi alloys [3]. The result here agrees with these values over the temperature range they studied. This emphasizes the importance of measurements at $T > 20$ K as well to obtain the true temperature dependence of the scattering mechanisms determining τ_{ie}. Bieri *et al* [5] have measured $\Delta\rho \sim B^2$ on two strong spin–orbit scattering systems, $Cu_{50}Ti_{50}$ and $Y_{80}Si_{20}$, when $0.13\text{ K} < T < 10$ K. For $Cu_{50}Y_{50}$ they found a possible saturation for τ_{ie} with $\tau_{ie}(T = 0) \simeq 100$ ps and $\tau_{ie} \sim T^{-0.5}$ for $T < 4.5$ K. The temperature dependence of τ_{ie} is weaker than for $Cu_{65}Ti_{35}$ and follows a $T^{-0.5}$ law very nicely. It is, however, not possible to explain this temperature behaviour with any present theory of τ_{ie}.

A power-law expression for τ_{ie} was found in 2D thin Cu films with an electron–phonon term $\sim A_2T^3$ [37]. Simple 2D metal films have been shown to be systems where the study of WL effects can be driven to excellence [15]. The inelastic scattering time thus seems to have a fairly universal temperature dependence.

5. Summary

Two methods of simplifying the analysis of weak localization and electron–electron interaction effects in the magnetoresistance of amorphous metals have been studied. The first suggests that it is possible to separate the weak localization contribution $\Delta\rho(B)_{WL}$ and electron–electron interaction contribution $\Delta\rho(B)_{EEI}$. After the separation one can determine the sample-dependent constants controlling $\Delta\rho(B)_{EEI}$ with less uncertainty than otherwise. Theory seems to describe this contribution correctly up to at least $\mu_B B/k_B T \simeq 35$. A good agreement between parameters from $\rho(T)$ and $\Delta\rho(B)_{EEI}$ is found. The second method studied the possibilities to get the inelastic scattering time τ_{ie} and the spin–orbit scattering time τ_{so} from the limit $\Delta\rho(B) \sim B^2$ in a strong spin–orbit scattering system. For $Cu_{65}Ti_{35}$ a weak temperature dependence $\tau_{ie} \sim T^{-1.15\pm0.1}$ below 5 K was observed and when $20 \text{ K} \leqslant T \leqslant 50 \text{ K}$ a strong temperature dependence with $\tau_{ie} \sim T^{-3.7\pm0.5}$. The behaviour of τ_{ie} is similar to those found in weak spin–orbit scattering systems and can be interpreted as a combination of a saturating τ_{ie}, inelastic electron–electron scattering and inelastic electron–phonon scattering. The electron–phonon scattering contribution has a strong temperature dependence $\tau_{ie} \sim T^{-n}$ with $n \geqslant 3$.

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